

General Certificate of Education  
January 2009  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Pure Core 2**

**MPC2**

Tuesday 13 January 2009 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
  - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

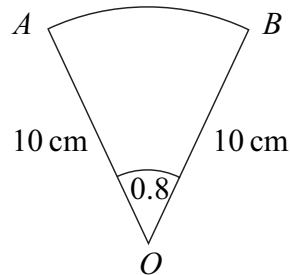
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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- 1 The diagram shows a sector  $OAB$  of a circle with centre  $O$  and radius 10 cm.



The angle  $AOB$  is 0.8 radians.

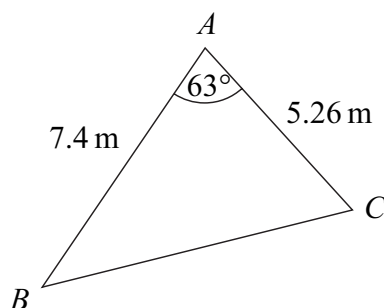
- (a) Find the area of the sector. (2 marks)
- (b) (i) Find the perimeter of the sector  $OAB$ . (3 marks)
- (ii) The perimeter of the sector  $OAB$  is equal to the perimeter of a square. Find the area of the square. (2 marks)
- 2 (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_{1.5}^6 x^2 \sqrt{x^2 - 1} \, dx$$

giving your answer to three significant figures. (4 marks)

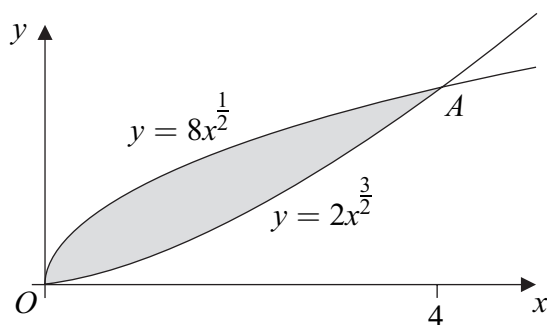
- (b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. (1 mark)

- 3 The diagram shows a triangle  $ABC$ .



The size of angle  $A$  is  $63^\circ$ , and the lengths of  $AB$  and  $AC$  are 7.4 m and 5.26 m respectively.

- (a) Calculate the area of triangle  $ABC$ , giving your answer in  $\text{m}^2$  to three significant figures. (2 marks)
- (b) Show that the length of  $BC$  is 6.86 m, correct to three significant figures. (3 marks)
- (c) Find the value of  $\sin B$  to two significant figures. (2 marks)
- 4 The diagram shows a sketch of the curves with equations  $y = 2x^{\frac{3}{2}}$  and  $y = 8x^{\frac{1}{2}}$ .



The curves intersect at the origin and at the point  $A$ , where  $x = 4$ .

- (a) (i) For the curve  $y = 2x^{\frac{3}{2}}$ , find the value of  $\frac{dy}{dx}$  when  $x = 4$ . (2 marks)
- (ii) Find an equation of the normal to the curve  $y = 2x^{\frac{3}{2}}$  at the point  $A$ . (4 marks)
- (b) (i) Find  $\int 8x^{\frac{1}{2}} dx$ . (2 marks)
- (ii) Find the area of the shaded region bounded by the two curves. (4 marks)
- (c) Describe a single geometrical transformation that maps the graph of  $y = 2x^{\frac{3}{2}}$  onto the graph of  $y = 2(x + 3)^{\frac{3}{2}}$ . (2 marks)

Turn over ►

- 5 (a) By using the binomial expansion, or otherwise, express  $(1 + 2x)^4$  in the form

$$1 + ax + bx^2 + cx^3 + 16x^4$$

where  $a$ ,  $b$  and  $c$  are integers.

(4 marks)

- (b) Hence show that  $(1 + 2x)^4 + (1 - 2x)^4 = 2 + 48x^2 + 32x^4$ .

(3 marks)

- (c) Hence show that the curve with equation

$$y = (1 + 2x)^4 + (1 - 2x)^4$$

has just one stationary point and state its coordinates.

(4 marks)

- 6 (a) Write each of the following in the form  $\log_a k$ , where  $k$  is an integer:

(i)  $\log_a 4 + \log_a 10$ ;

(1 mark)

(ii)  $\log_a 16 - \log_a 2$ ;

(1 mark)

(iii)  $3 \log_a 5$ .

(1 mark)

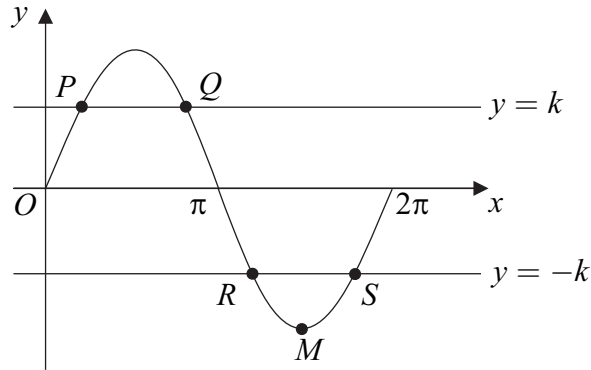
- (b) Use logarithms to solve the equation  $(1.5)^{3x} = 7.5$ , giving your value of  $x$  to three decimal places.

(3 marks)

- (c) Given that  $\log_2 p = m$  and  $\log_8 q = n$ , express  $pq$  in the form  $2^y$ , where  $y$  is an expression in  $m$  and  $n$ .

(3 marks)

- 7 (a) Solve the equation  $\sin x = 0.8$  in the interval  $0 \leq x \leq 2\pi$ , giving your answers in radians to three significant figures. (3 marks)
- (b) The diagram shows the graph of the curve  $y = \sin x$ ,  $0 \leq x \leq 2\pi$  and the lines  $y = k$  and  $y = -k$ .



The line  $y = k$  intersects the curve at the points  $P$  and  $Q$ , and the line  $y = -k$  intersects the curve at the points  $R$  and  $S$ .

The point  $M$  is the minimum point of the curve.

- (i) Write down the coordinates of the point  $M$ . (2 marks)
- (ii) The  $x$ -coordinate of  $P$  is  $\alpha$ .  
Write down the  $x$ -coordinate of the point  $Q$  in terms of  $\pi$  and  $\alpha$ . (1 mark)
- (iii) Find the length of  $RS$  in terms of  $\pi$  and  $\alpha$ , giving your answer in its simplest form. (2 marks)
- (c) Sketch the graph of  $y = \sin 2x$  for  $0 \leq x \leq 2\pi$ , indicating the coordinates of points where the graph intersects the  $x$ -axis and the coordinates of any maximum points. (5 marks)

- 8 The 25th term of an arithmetic series is 38.

The sum of the first 40 terms of the series is 1250.

- (a) Show that the common difference of this series is 1.5. (6 marks)
- (b) Find the number of terms in the series which are less than 100. (3 marks)

**END OF QUESTIONS**

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